Cross Phase Modulation Enhancement by Coupled Cavities and Electromagnetically Induced Transparency

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Abstract

We propose an optical double-cavity resonator whose response to a signal is similar to that observed in electromagnetically induced transparency (EIT). A combination of such a device with a four-level EIT medium can serve for achieving large cross-Kerr modulation of a probe field by a signal field.

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1. Introduction

Making a single photon to strongly influence another photon has been an exciting challenge to quantum optics. Such an effect would have rich applications, e.g., in quantum information processing [1]. A possible approach is to use a large cross-Kerr nonlinearity in EIT [2]. The signal field can then induce a phase shift of the probe field. However, for achieving a noticeable effect by a single signal photon, it is crucial (i) to concentrate the signal photon into a small volume so that its electric field is large, and (ii) to ensure long interaction time of the signal and probe fields.

One possible way to meet these conditions is to let the signal and probe interact inside a common cavity. However, the combination of the cavity properties with the ultra-low group velocity of light in the EIT medium leads to an extremely narrow line of the transmitted light, which introduces some limits of the applications that are possible in principle [3]. Another way is to work with short pulses (thus confining the photons into a small volume) and to slow down the signal pulse by an additional EIT medium [4]. Thus, a long interaction time can be achieved for pulses which otherwise would travel with very different group velocities. However, most of the signal-photon energy is then wasted dressing the atoms of the (second) EIT medium [5], so that only a small fraction \( \approx v_R/c \) of it is available for shifting the phase of the probe.

Here we discuss a different method for satisfying the conditions (i) and (ii). The signal propagates through a special double-cavity resonator whose response is analogous to that of an EIT medium. Thus, a relatively strong field can be built in one cavity for a reasonably long time. The probe propagates across the cavity, perpendicularly to its axis, through the EIT medium. Thus, the probe feels a strong EIT effect, but is not influenced by the cavity. We discuss the experimental requirements for achieving this effect.

2. Double-cavity Resonator

The EIT is an interference effect where a probe field resonant with an atomic transition freely propagates through the medium [see Fig. 1(a)]. In a narrow transparency window the
Fig. 1. (a) Scheme of the EIT: probe with frequency $\omega_p$ propagates through a medium with a resonant transition $|1\rangle \rightarrow |3\rangle$, the excited level $|3\rangle$ being coupled to the level $|2\rangle$ with a strong coupling field with frequency $\omega_c$ and Rabi frequency $\Omega$. (b) Scheme of the double-cavity system. The signal enters (a) and leaves (a') the system via the beam splitter BS$_1$ which couples the vertical cavity with the external world. The horizontal cavity is coupled to the vertical one with the beam splitter BS$_2$.

Fig. 2. (a) Response function $G_{11}$ for $L_V = 120\lambda_0$, $L_H = 30\lambda_0$, $R_1 = 10\%$, and $R_2 = 10^{-6}$ ($\lambda_0 = 2\pi c/\omega_0)$. Dashed line: Re $G_{11}$, full line: Im $G_{11}$. (b) Time evolution of the energy fraction in the space in front of the system (broken line), inside the horizontal cavity (full line), and in the space behind the system (dash-dotted line), and relative intensities entering and leaving the system at BS$_1$ (dotted lines). The input pulse is a Gaussian with a half width $4\tau_D$. 

$\omega_c, \Omega$
probe field is subject to a specific dispersion relation which causes extremely slow pulse propagation [6]. An analogous interference effect can also be observed in a specially designed double-cavity resonator.

Let us consider two cavities of lengths $L_V$ and $L_H$ coupled to each other by a weakly reflecting beam splitter of reflectivity $R_2$ [see Fig. 1(b)]. The vertical cavity is coupled to the outside world by another weakly reflecting beam splitter of reflectivity $R_1$. The (c-number) input-output relations for the fields $a$, $b$ (input) and $a'$, $b'$ (output) can be written as

$$
\begin{pmatrix}
    a' \\
    b'
\end{pmatrix} =
\begin{pmatrix}
    G_1 & G_{12} \\
    G_{21} & G_{22}
\end{pmatrix}
\begin{pmatrix}
    a \\
    b
\end{pmatrix},
$$

where the explicit form of the $G$ coefficients is given in [7]. The typical dependence on frequency of $G_{11}(\omega)$ near resonance is illustrated in Fig. 2(a). Resonant light of frequency $\omega_0 = n_V\pi c/L_V = n_H\pi c/L_H$ ($n_{V,H}$ being integers) is transmitted, whereas nearby frequency components $\omega \pm \Delta \omega_0$ with $\Delta \omega = c\sqrt{R_2/(L_H L_V)}$ are reflected. Note the similarity of $G_{11}(\omega)$ to the shape of the dispersion relation of an EIT medium (see, e.g., [6]). As a result, an incoming pulse of central frequency $\omega_0$ is delayed by $\tau_D = R_1 L_H/(2R_2 c)$ [see Fig. 2(b)], and a relatively strong field is built inside the horizontal cavity.

The analogy between an EIT medium [Fig. 1(a)] and a double-cavity system [Fig. 1(b)] can be understood as follows. The empty double-cavity system corresponds to the atomic (ground) state $|1\rangle$ and light in the vertical cavity corresponds to the excited atomic state $|3\rangle$. Accordingly, light in the horizontal cavity corresponds to the (auxiliary) atomic state $|2\rangle$. Coupling of the atomic states $|3\rangle$ and $|2\rangle$, which gives rise to the dressed (Rabi-split) states, is analogous to the coupling of the cavities by the beam-splitter BS$_2$.

### 3. Double-cavity Resonator and Four-level EIT

The relatively strong field and the long time spent by the field in the horizontal cavity can be used for inducing a phase shift of a probe by a signal in a four-level EIT scheme. Let us assume the level structure in Fig. 3(a). Using fourth-order perturbation theory, one can find

![Fig. 3.](image)

(a) Four-level EIT scheme [2]; the signal field with frequency $\omega_0$ is detuned by $\Delta$ of the transition $|2\rangle \leftrightarrow |4\rangle$ and influences the phase shift of the probe with frequency $\omega_p$, resonant with the $|1\rangle \leftrightarrow |3\rangle$ transition. The levels $|2\rangle$ and $|3\rangle$ are coupled with a strong coupling field. (b) Combined double-cavity resonator with a four-level EIT medium. The horizontal cavity is filled with an EIT medium. The signal propagates as in Fig. 1(b). The probe propagates across the horizontal cavity in the direction perpendicular to the plane of the plot.
that the Kerr index of refraction felt by the probe is

\[ n_K = \frac{N\mu_{13}^2\mu_{24}^2}{8\epsilon_0\hbar^3 |\Omega|^2 \Delta} |E_s|^2, \]  

(2)

where \( N \) is the atomic density, \( \mu_{13} \) and \( \mu_{24} \) are respectively the dipole matrix elements of the transitions \( |1\rangle \leftrightarrow |3\rangle \) and \( |2\rangle \leftrightarrow |4\rangle \), \( \Omega \) is the coupling field Rabi frequency, \( \Delta \) is the signal detuning, and \( E_s \) is the signal electric field strength. The group velocity \( v_g \) of the probe can be given by \[ v_g = \frac{c\left(n_0 + \omega_p \frac{dn_0}{d\omega_p}\right)^{-1}}{4\pi N\mu_{13}^2}, \]

(3)

where \( n_0 \) is the signal-independent part of the refractive index.

Let us assume that the signal and the probe are sent, according to Fig. 3(b), to the double-cavity system complemented with the four-level EIT medium. Note that the probe propagates perpendicularly to the cavity axis. The conditional phase shift \( \Delta \phi \) of the probe due to cross-phase modulation is

\[ \Delta \phi = \int dl n_K k_p, \]

(4)

where \( k_p = \omega_p / c \), and the integral runs over the propagation length of the probe. For a pulse propagating with the group velocity \( v_g \) in an otherwise homogeneous medium, we may write \( dl = v_g dt \) and use Eqs. (4) and (5), so that

\[ \Delta \phi = \frac{\mu_{24}^2}{16\hbar^2 \Delta} \int dt |E_s(t)|^2. \]

(5)

The integral runs over the time during which the probe pulse is inside the medium. Let the input signal be a single-photon Gaussian pulse of half-width \( \tau_s \) and cross-sectional area \( S \). When the probe group velocity is sufficiently low, so that the time which the probe spends in the medium is longer than the time duration of the signal in the cavity, \( L/v_g > 4 \sqrt{\tau_s^2 + \tau_D^2}/2 \) (\( L \) being the propagation length in the EIT medium), then the time integral in Eq. (7) can be approximated by shifting the limits to infinity, and the phase shift of the probe is estimated to be \[ \Delta \phi \approx \frac{\pi}{8} \frac{R_1}{R_2} \frac{\mu_{24}^2}{\epsilon_0\hbar\lambda S\Delta} \left(1 + \frac{\tau_D^2}{2\tau_s^2}\right)^{-1/2}. \]

(6)

Now, the central question is that of the experimental conditions under which this phase shift can attain values of \( \approx \pi \).

4. Experimental Conditions

For achieving large phase shifts by a single photon, one must find a balance between different requirements. \( i \) The probability \( P_2 \) of a two-photon absorption (signal and probe) must be small, i.e., \[ \frac{P_2}{\Delta \phi} \approx \frac{2\pi \gamma_4}{\Delta} \ll 1, \]  

(7)
where $\gamma'_4$ is the atomic decay rate of the state $j_4$. (ii) The single-photon absorption of the probe in the EIT medium must be small which limits the spectral width $\delta \approx \tau_p^{-1}$ of the probe pulse for given atomic density,

$$\frac{32\pi^2 NL\mu^2_{13}\gamma_3\delta^2}{\epsilon_0\hbar|\Omega|^4} \ll 1. \tag{8}$$

(iii) The time duration $\tau_p \approx \delta^{-1}$ of the probe must be shorter than its propagation time through the medium. From Eq. (5) it follows that this assumption yields the condition that

$$\frac{1}{\delta} \ll \frac{L}{v_g} = \frac{4\pi NL\mu^2_{13}}{\epsilon_0\hbar|\Omega|^2}. \tag{9}$$

By combining the conditions (8) and (9), we obtain a condition for the atomic density,

$$N \gg \frac{2\epsilon_0\hbar\gamma_3}{\mu^2_{13}L}, \tag{10}$$

and a condition for the line-width of the probe,

$$\delta \ll \frac{|\Omega|^2}{8\pi\gamma_3}. \tag{11}$$

From Eq. (8) it follows that the ratio $R_1/R_2$ should be chosen as large as possible.

In the experimental demonstration of ultraslow group velocity in Ref. [8] a gas of rubidium atoms was used. In this case, $\lambda = 795$ nm, $\mu_{13} \approx \mu_{24} \approx 10^{-25}$ Cm, and $\gamma_3 \approx \gamma_4 \approx 10^6$ s$^{-1}$. Let the signal beam diameter be $\approx L_H \approx 10$ $\mu$m so that $S \approx 10^{-10}$ m$^2$. The condition (7) is satisfied, $P_2/\Delta \phi \approx 0.1$, if $\Delta = 10^8$ s$^{-1}$. From Eq. (8) it then follows that for $R_1/R_2 \approx 10^5$ a phase shift of $\Delta \phi \approx \pi$ can be achieved.

The condition (10) requires that $N \gg 10^{12}$ cm$^{-3}$. Let us assume an atomic density of $N \approx 10^{14}$ cm$^{-3}$. Since the detuning $\Delta$ is very small, the gas has to be laser-cooled to avoid Doppler broadening. To achieve a sufficiently small group velocity of the probe, the Rabi frequency of the driven atomic transition should be $|\Omega| \approx 10^9$ s$^{-1}$. With these values we find from Eqs. (2) and (3) for the group velocity $v_g \approx 10^3$ ms$^{-1}$, the switching time being $L_H/v_g \approx 10$ ns. According to the condition (11), the length of the probe pulse should be chosen such that $\tau_p \gg 10^{-11}$ s. A reasonable choice (ensuring relatively small absorption) appears to be $\tau_p \approx 1$ ns (see [7] for more details).

5. Conclusion

The most important features of the proposed scheme are as follows. The cavity system emulates the EIT response for the signal: the signal is slowed down and simultaneously its intensity is enhanced. The probe is not influenced by the cavity, but it propagates in the EIT medium whose properties are determined by the signal.

To achieve a large ($\approx \pi$) phase-shift by means of a single signal photon, several technical problems should be solved. As the most important, very low-reflectivity ($R < 10^{-6}$) beam-splitters must be available and a suitable method for keeping the cooled EIT atoms in an optical cavity must be used. The device then could serve as a relatively fast ($\approx 10$ ns switching time) quantum gate.
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References