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Conditional Schrödinger-cat output states of a beam splitter[1]

M. Dakna[2], T. Opatrný, L. Knöll and D.–G. Welsch
Friedrich-Schiller-Universität Jena, Theoretisch-Physikalisches Institut
Max-Wien-Platz 1, D-07743 Jena, Germany

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We show that Schrödinger cat-like states can be generated by means of conditional measurements using a simple beam splitter scheme. When a squeezed vacuum and an ordinary vacuum are mixed and in one of the output channels the number of photons is measured, then conditional quantum states in the other output channel can be obtained which reveal all the properties of superpositions of macroscopically distinguishable states. We analyze the conditional states in terms of the photon-number and quadrature-component distributions and the Wigner and Husimi functions and present both analytical and numerical results. We also study the component states and show that they are very close to squeezed coherent states and approach coherent states for sufficiently large numbers of detected photons. Further, we consider the problem of producing the superposition states under the conditions of realistic photocounting. For this purpose we assume multichannel detection using highly efficient avalanche photodiodes. As expected, the measurement smears the interference structure. However, for properly chosen parameters the interference structure can still be found even for a realistic arrangement of detectors.

1. Introduction

The basic difference between classical and quantum mechanics is that the later allows according to the superposition principle the phenomenon of quantum interference of two complex wave amplitudes. The experimental tests of this principle has been an challenge since the early days of quantum mechanics. Recently the developments in quantum optics have provides possibilities for observing superposition states i.e., Schrödinger-cat like states[1] of a trapped ion [2-4]. In the realm of traveling optical fields the up to now proposals for the generation such a state have been found to be very difficult to implement experimentally. Therefore schemes for generating such a states are still of

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[2]E-mail address: dakna@tpi.uni-jena.de

considerable interest. A promising way for the generation of the Schrödinger-cat states is the use of the state reduction via conditional quantum measurements. According to basic ideas of the quantum theory of measurement, if a quantity of a subsystem of a correlated two-part system is measured, the state of the other subsystem collapses to a particular state. Inspired by this facts several ingenious schemes to produce conditional states of the type of Schrödinger cat-like states have been proposed [6-9] and their experimental feasibilities with current technologies have been recently reported [10]. In this paper we propose a relative simple scheme for generating Schrödinger cat-like states by means of conditional measurements on a beam splitter [11]. It is showed that when a squeezed vacuum enters one of the input channels (the second input channel being unused) and the photon number of the mode in one of the output channels is measured, then the conditional state of the signal mode in the other output channel shears all the typical features of a Schrödinger cat-like state. The present paper is organized as follows. Next we calculate the conditional output states and investigate their properties. We then discuss the effect a realistic detection scheme. A summary is given finally.

2. The conditional output states

It is well known that the input-output relations of a beam splitter, the most commonly used element in quantum optical experiments may be characterized by the SU(2) algebra. In the Schrödinger picture of quantum mechanics, the output density operator $\hat{\varrho}_{\rm out}$ of this device is obtained from the input density operator $\hat{\varrho}_{\rm in}$ as $\hat{\varrho}_{\rm out} = \hat{V}^\dagger \hat{\varrho}_{\rm in} \hat{V}$. In this relation and henceforth we will ignore the phase shift caused by the beam splitter. since such a phase shift does not affect our results. The operator $\hat{V}$ is given by [12] $\hat{V} = e^{-2i\theta\hat{L}_2}$, with $\hat{L}_2 = \frac{1}{2i}(\hat{a}_1^\dagger\hat{a}_2 - \hat{a}_2^\dagger\hat{a}_1)$. Now, let us consider that the input-state density operator have the form

$$\hat{\varrho}_{\rm in} = \hat{\varrho}_{\rm in1} \otimes |{\rm vac}_2\rangle\langle{\rm vac}_2|. \tag{1}$$

Then the output-state density operator can be given by [11]

$$\hat{\varrho}_{\rm out} = \sum_{n_2=0}^{\infty}\sum_{m_2=0}^{\infty} \frac{(-1)^{m_2+n_2}}{\sqrt{m_2! n_2!}} \left|\frac{R}{T}\right|^{m_2+n_2} \hat{a}_1^{m_2} |T|^{\hat{a}_1^\dagger\hat{a}_1} \hat{\varrho}_{\rm in1} |T|^{\hat{a}_1^\dagger\hat{a}_1} (\hat{a}_1^\dagger)^{n_2} \otimes |m_2\rangle\langle n_2|, \tag{2}$$

where $|T| = \cos\theta$ and $|R| = \sin\theta$, and $|n_k\rangle$ are the eigenstates of the photon-number operators $\hat{a}_k^\dagger\hat{a}_k$. From Eq. (2) we see that the output modes are, in general, highly correlated. When the photon number of the mode in the second output channel is measured and m_2 photons are detected, then the mode in the first output channel is prepared in a quantum state whose density operator $\hat{\varrho}_{\rm out1}$ reads as

$$\hat{\varrho}_{\rm out1}(m_2) = \frac{\langle m_2|\hat{\varrho}_{\rm out}|m_2\rangle}{{\rm Tr}_1(\langle m_2|\hat{\varrho}_{\rm out}|m_2\rangle)}. \tag{3}$$

The probability of such an event is given by

$$P(m_2) = {\rm Tr}_1(\langle m_2|\hat{\varrho}_{\rm out}|m_2\rangle) = \sum_{n_1=m_2}^{\infty} \binom{n_1}{m_2} \left(1-|T|^2\right)^{m_2} |T|^{2(n_1-m_2)} \langle n_1|\hat{\varrho}_{\rm in1}|n_1\rangle. \tag{4}$$

In particular, if the first input mode is prepared in a squeezed vacuum state, we may write $\hat{\varrho}_{\text{in}1} = \hat{S}(\xi)|\text{vac}_1\rangle\langle\text{vac}_1|\hat{S}^\dagger(\xi)$, where

$$\hat{S}(\xi)|\text{vac}_1\rangle = \exp\left\{-\tfrac{1}{2}\left[(\xi\hat{a}_1^\dagger)^2 - \xi^*\hat{a}_1^2\right]\right\}|\text{vac}_1\rangle = (1-|\kappa|^2)^{\frac{1}{4}} \sum_{n_1=0}^{\infty} \frac{\sqrt{[2n_1]!}}{2^{n_1} n_1!} \kappa^{n_1} |2n_1\rangle, \quad (5)$$

$\kappa = \tanh|\xi|$. Combining Eqs. (2) and (3) and using Eqs. (1), and (5), we obtain

$$\hat{\varrho}_{\text{out}1}(m_2) = |\Psi_{m_2}\rangle\langle\Psi_{m_2}|, \quad (6)$$

where

$$|\Psi_{m_2}\rangle = |\Psi_{m_2}(\alpha)\rangle = \frac{1}{\sqrt{\mathcal{N}_{m_2}}} \sum_{n_1=0}^{\infty} c_{m_2,n_1}(\alpha)\,|n_1\rangle, \quad (7)$$

$$c_{m_2,n_1}(\alpha) = \frac{(n_1+m_2)!}{\Gamma\left[\frac{1}{2}(n_1+m_2)+1\right]\sqrt{n_1!}} \frac{1}{2}\left[1+(-1)^{n_1+m_2}\right]\left(\tfrac{1}{2}\alpha\right)^{\frac{1}{2}(n_1+m_2)}, \quad (8)$$

with $|\alpha| = |T|^2\kappa$ the normalization constant can be given by [9]

$$\mathcal{N}_{m_2} = \sum_{n_1=0}^{\infty} |c_{m_2,n_1}(\alpha)|^2 = \frac{1}{\sqrt{1-\alpha^2}} \left[\frac{\alpha^2}{(1-\alpha^2)}\right]^{m_2} \sum_{k=0}^{\left[\frac{1}{2}m_2\right]} \frac{(m_2!)^2}{(m_2-2k)!\,(k!)^2\,(2\alpha)^{2k}}, \quad (9)$$

($[x]$ denotes the integral part of x). The probability $p(m_2)$ may be given by [9]

$$P(m_2) = \sqrt{\frac{1-\kappa^2}{1-\alpha^2}} \left[\frac{\alpha^2(1-|T|^2)}{|T|^2(1-\alpha^2)}\right]^{m_2} \sum_{k=0}^{\left[\frac{1}{2}m_2\right]} \frac{m_2!}{(m_2-2k)!(k!)^2(2\alpha)^{2k}}. \quad (10)$$

¿From Eqs. (7) and (8) we easily see that when the detected number of photons in one output channel, m_2, is even (odd), then the mode in the other output channel is prepared in a quantum state $|\Psi_{m_2}\rangle$ that contains only contributions from photon-number states with even (odd) numbers of photons. In particular, when the number of detected photons, m_2, is zero, then the output state is again a squeezed vacuum, but with the parameter α in place of κ.

We now study the properties of the conditional states $|\Psi_m\rangle$ in more detail. For notational convenience we will omit the subscripts 1 and 2 introduced above to distinguish between the two output channels. Thus the photon-number distribution of a state $|\Psi_m\rangle$ reads as

$$P(n|m) = |\langle n|\Psi_m\rangle|^2 = \mathcal{N}_m^{-1}\,|c_{m,n}(\alpha)|^2\,, \quad (11)$$

where $c_{m,n}(\alpha)$ and $\mathcal{N}_m$ are given in Eqs. (8) and (9), respectively. In particular, the mean photon number can be given by

$$\langle\hat{n}\rangle = \alpha\frac{\partial}{\partial\alpha}\log\left[\frac{\mathcal{N}_m}{\alpha^m}\right]\frac{\alpha^2}{1-\alpha^2} + m\frac{1+\alpha^2}{1-\alpha^2} - 2\sum_{k=0}^{\left[\frac{1}{2}m\right]} k\,a_{k,m}\left[\sum_{k=0}^{\left[\frac{1}{2}m\right]} a_{k,m}\right]^{-1}, \quad (12)$$

where $a_{k,m} = (2\alpha)^{-2k}/[(m-2k)!\,(k!)^2]$. Note that when no photons are detected, $m=0$, then $\langle\hat{n}\rangle$ reduces to the mean number of photons of a squeezed vacuum, $\langle\hat{n}\rangle = \alpha^2/(1-\alpha^2)$. The conditional quadrature-component distribution $p(x,\varphi|m) = |\langle x,\varphi|\Psi_m\rangle|^2$, the quantity which can be measured in balanced homodyne detection reads [9]

$$p(x,\varphi|m) = \frac{|\alpha|^m}{\mathcal{N}_m\sqrt{\pi\Delta^{m+1}}\,2^m}\exp\left(-\frac{1-\alpha^2}{\Delta}\,x^2\right)\left|\mathrm{H}_m\left[\sqrt{(\alpha e^{i2\varphi}-\alpha^2)/\Delta}\,x\right]\right|^2, \tag{13}$$

where the abbreviation $\Delta = 1+\alpha^2-2\alpha\cos(2\varphi)$ has been used. For φ near $\pi/2$ the quadrature-component distribution $p(x,\varphi|m)$ exhibits two separated peaks, whereas for φ close to 0 or π an interference pattern is observed (cf. the figure). This is one of the typical signatures of the superposition of two macroscopically distinguishable states [13]. The Wigner function $W(x,p|m)$ of the state $|\Psi_m\rangle$ reads [9]

$$W(x,p|m) = \frac{\exp\left(-\lambda x^2-\frac{p^2}{\lambda}\right)}{\pi\mathcal{N}_{1m}}\sum_{k=0}^{m}\frac{(-2|\alpha|)^k}{k![(m-k)!]^2}\left|\mathrm{H}_{m-k}\left[i\sqrt{\alpha\lambda}\,\left(x+i\frac{p}{\lambda}\right)\right]\right|^2, \tag{14}$$

where

$$\lambda = \frac{1-\alpha}{1+\alpha}, \quad \mathcal{N}_{1m} = \sum_{k=0}^{[\frac{1}{2}m]}\frac{(2|\alpha|)^{m-2k}}{(m-2k)!\,(k!)^2}\,. \tag{15}$$

Whereas the Husimi function $Q(x,p|m)$ of the state $|\Psi_m\rangle$ is found to be [9]

$$Q(x,p|m) = \frac{|\alpha|^m}{\pi\mathcal{N}_m 2^{m+1}}\left|\mathrm{H}_m\left[\tfrac{1}{2}i\sqrt{\alpha}(x+ip)\right]\right|^2\exp\left\{-\tfrac{1}{2}\left[(1-\alpha)x^2+(1+\alpha)p^2\right]\right\}. \tag{16}$$

¿From Eqs. (7) and (8) it is seen that $|\Psi_m\rangle$ can be regarded as a superposition of states $|\Psi_m^{(+)}\rangle$ and $|\Psi_m^{(-)}\rangle$ as $|\Psi_m\rangle = A\left(|\Psi_m^{(+)}\rangle + |\Psi_m^{(-)}\rangle\right)$, where

$$|\Psi_m^{(\pm)}\rangle = \frac{1}{\sqrt{\mathcal{N}_m^{(\pm)}}}\sum_{n=0}^{\infty}c_{m,n}^{(\pm)}(\alpha)|n\rangle, \tag{17}$$

with

$$c_{m,n}^{(\pm)}(\alpha) = \frac{(n+m)!}{\Gamma[(n+m)/2+1]\sqrt{n!}}\left(\pm\sqrt{\tfrac{1}{2}\alpha}\right)^{n+m}. \tag{18}$$

The normalization factor $\mathcal{N}_m^{(\pm)}$ is calculated to be [9]

$$\mathcal{N}_m^{(\pm)} = 2\mathcal{N}_m - \frac{m!^2(-|\alpha|)^m}{\sqrt{\pi}2^m\Gamma(m+3/2)}\mathrm{F}\left[\tfrac{1}{2}(m+1),\tfrac{1}{2}(m+1),\tfrac{1}{2}(2m+3),1-\alpha^2\right], \tag{19}$$

$\mathcal{N}_m$ being given in Eq. (9) and $\mathrm{F}(\alpha,\beta,\gamma,z)$ stands for the hypergeometric function. A deeper inspection of the states $|\Psi_m^{(\pm)}\rangle$ for example in term of their Wigner functions reveals that they are very close to squeezed coherent states [9].

5. Realistic photon counting

In order to produce the conditional states $|\Psi_m\rangle$, highly efficient and precise photocounting is needed. To this aim we consider here the so called photon chopping method [17] and assume non perfect detection. Using a multichannel detection device [17] the probability of recording k coincident events is given by

$$\tilde{P}_{N,\eta}(k|m) = \sum_l \tilde{P}_N(k|l)\, M_{l,m}(\eta), \tag{20}$$

where $\tilde{P}_N(k|m)$ and $M_{l,m}(\eta)$ are respectively given by

$$\tilde{P}_N(k|m) = \frac{1}{N^m}\binom{N}{k}\sum_{l=0}^{k}(-1)^l\binom{k}{l}(k-l)^m, \text{ for } k \leq m, \text{ and } \tilde{P}_N(k|m) = 0, \text{ for } k > m$$

$$M_{l,m}(\eta) = \binom{m}{l}\eta^l(1-\eta)^{m-l}, \text{ for } l \leq m, \text{ and } M_{l,m}(\eta) = 0, \text{ for } l > m \tag{21}$$

Since detection of k coincident events can result from various numbers m of photons, the conditional state is in general a statistical mixture. Therefore in place of Eq. (6) we now have

$$\hat{\varrho}_{\mathrm{out}}(k) = \sum_m P_{N,\eta}(m|k)\,|\Psi_m\rangle\langle\Psi_m|, \tag{22}$$

where the conditional probability $P_{N,\eta}(m|k)$ can be obtained using the Bayes rule as

$$P_{N,\eta}(m|k) = \frac{1}{\tilde{P}_{N,\eta}(k)}\tilde{P}_{N,\eta}(k|m)\;P(m), \quad \text{with} \quad \tilde{P}_{N,\eta}(k) = \sum_m \tilde{P}_{N,\eta}(k|m)\;P(m) \tag{23}$$

Here $P(m)$ is the prior probability (10) of m photons being present, and accordingly, $\tilde{P}_{N,\eta}(k)$ is the prior probability of recording k coincident events,

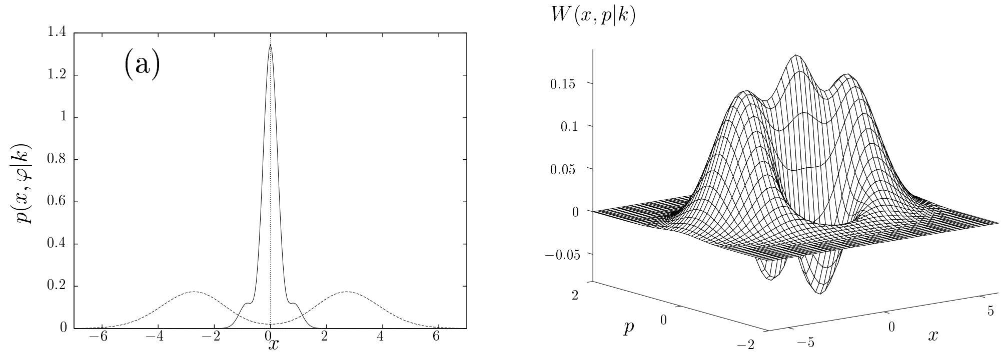

Fig.1. Quadrature distribution (a) [for the phase parameters $\varphi=0$ (full line) and $\varphi=\pi/2$ (broken line)] and the Wigner function (b) of the conditional state with realistic photodetection ($N=10$, $\eta=0.8$) for $k=2$ coincident events, and $\kappa = 0.75$, $|T|^2 = 0.8$ ($\alpha = 0.6$).

6. Conclusions

The feasibility of generating Schrödinger cat-like states has via conditional measurements on a beam splitter have been studied. We showed that when a squeezed vacuum and an ordinary vacuum are mixed by a beam splitter and in one of the output channels the number of photons is measured, then the conditional quantum state in the other output channel reveals all the properties of a Schrödinger cat-like state. Analytical results of the photon-number and quadrature-component distributions and the Wigner and Husimi functions of the conditional states have been given. We have expressed the conditional states as superposition of two quasiclassical component states that are very close to squeezed coherent states and approach coherent states for sufficiently large numbers of detected photons. Assuming multichannel detection using highly efficient avalanche photodiodes. We have also discussed the problem of generating the Schrödinger cat-like states under the conditions of realistic photocounting. We found that the measurement smears the interference structure. However, the interference structure can still be found even for a realistic arrangement of detectors.

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